On Fractional Instanton Numbers in Six Dimensional Heterotic $\mathbf{E}_8 \times \mathbf{E}_8$ Orbifolds

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Abstract: We show how the level matching condition in six dimensional, abelian and supersymmetric orbifolds of the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string can be given equivalently in terms of fractional gauge and gravitational instanton numbers. This relation is used to restate the classification of the orbifolds in terms of flat bundles away from the orbifold singularities under the constraint of the level matching condition. In an outlook these results are applied to Kaluza-Klein monopoles of the heterotic string on S^1 in Wilson line backgrounds.

1 Introduction

In this talk we present some results of [16] and apply them to Kaluza-Klein monopoles of the heterotic string on S^1.

Even though heterotic orbifolds have been known for a long time [1, 2], their strong coupling description in terms of M-theory on $S^1/\mathbf{Z}_2$ is still very limited [10, 11, 12, 13, 14, 15]. Here we focus on symmetric, abelian and perturbative orbifolds of the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string in six dimensions preserving 8 supersymmetries without discrete torsion.

2 The level matching condition

At first, we consider a single fixed point in an arbitrary model located at the origin of $\mathbf{C}^2$. Associated to it is a generator (r, γ) of $\mathbf{Z}_N$ consisting of a rotation r acting like $\exp(2\pi i\Phi_i)$ on the complex coordinate Z^i and a gauge shift γ acting like $\exp(2\pi i\beta_1^I H_I^1) \exp(2\pi i\beta_2^I H_I^2)$ where the $H_I^{1,2}$ are the eight generators of the Cartan subalgebra in the adjoint representation of $\mathrm{E}_8^{(1,2)}$. The H_I will be normalized such that the E_8 lattice Λ_8 is given by the vectors $p^I = (n_1, \ldots, n_8)$ and $p^I = (\frac{1}{2}+n_1, \ldots, \frac{1}{2}+n_8)$ with $n_i \in \mathbf{Z}$ and $\sum_i n_i \in 2\mathbf{Z}$. This implies that $q_{1,2}^I = N\beta_{1,2}^I$ is a lattice vector.

Consistency of the model requires the level matching condition

$$\Phi^2 = \beta_1^2 + \beta_2^2 \mod \frac{2}{N} \tag{1}$$

to be satisfied [2, 6]. This equation, however, bears great similarity to the relation

$$\operatorname{tr} R^2 = \frac{1}{30} \operatorname{tr} F_1^2 + \frac{1}{30} \operatorname{tr} F_2^2 \tag{2}$$

required by the Green-Schwarz mechanism [7] together with perturbativity of the orbifold. Therefore one is naturally led to the question of to what extent the two equations are related. But already in the case of the simplest $\mathbf{Z}_3$ orbifolds it can be seen that (1) can not account for the full instanton numbers. To this end, consider the standard embedding $(1, -1, 0^6; 0^8)$ with the gauge group $U(1) \times E_7 \times E_8$ and compare to the embedding $(1, -1, 0^6; 2, 1^2, 0^5)$ with the gauge group $U(1) \times E_7 \times SU(3) \times E_6$. In the first case the instanton number for the second E_8 is zero whereas in the second case it cannot be zero!

However, by comparing to the $\operatorname{Spin}(32)/\mathbf{Z}_2$ case [3, 4] one can easily guess [5] that the level matching condition (1) corresponds to the formula

$$-\frac{1}{2} \frac{1}{8\pi^2} \int_U \operatorname{tr} R^2 = -\frac{1}{60} \frac{1}{8\pi^2} \int_U \operatorname{tr} F_1^2 - \frac{1}{60} \frac{1}{8\pi^2} \int_U \operatorname{tr} F_2^2 \pmod 1 \tag{3}$$

As was shown in [16], the fractional part of the instanton number for E_8 is indeed given by

$$I = -\frac{1}{60} \frac{1}{8\pi^2} \int_U \operatorname{tr} F^2 = \frac{N}{2} \beta^2 \pmod 1 \tag{4}$$

Note that this formula is clearly invariant under the Weyl group as well as under lattice shifts $\beta \mapsto \beta + p$, since

$$I = \frac{N}{2}(\beta + p)^2 = \frac{N}{2}(\beta^2 + 2pq/N + p^2) = \frac{N}{2}\beta^2 + pq + N\frac{p^2}{2} = \frac{N}{2}\beta^2 \pmod 1 \tag{5}$$

and $pq \in \mathbf{Z}$, $p^2 \in 2\mathbf{Z}$.

Therefore the level matching condition in the form of (3) corresponds to the condition that at every fixed point the sum of the fractional parts of the gauge instanton numbers match the fractional part of the gravitational instanton number (which is $1/N$). In particular the level matching condition as given in (1) has nothing to say on how the integer part of the gauge instanton numbers gets distributed among the two E_8 groups, i.e. the two walls in M-theory on $S^1/\mathbf{Z}_2$.

3 Global description of the orbifold

We now turn to the global description of the orbifold. The target space, i.e. the geometric orbifold, is conveniently described as $O = \mathbf{C}^2/S$ where S, the space group, is generated by affine transformations $D : x \mapsto rx + h$ [2] (for an introduction see

[8] or [9], chapter 16). As we restrict to abelian orbifolds, the point group, i.e. the group generated by the rotations r alone, is abelian.

To include the gauge degrees of freedom we augment D by a gauge transformation γ_D and require that the map $\gamma : S \rightarrow \mathrm{E}_8 \times \mathrm{E}_8$ is a group homomorphism. This implies that, when a nontrivial group element is associated to a pure translation h, it generates a cyclic subgroup of E_8. In this case the model is said to contain "discrete Wilson lines".

As we will now show, the map γ is nothing but the data of a flat $\mathrm{E}_8 \times \mathrm{E}_8$ bundle on the orbifold with the fixed points taken out. Define F to be the set of fixed points of elements of S. Then the fundamental group of $\mathbf{C}^2 - F$ is trivial since, by supersymmetry, F consists of isolated points only. Therefore $\mathbf{C}^2 - F$ is the universal covering space of $(\mathbf{C}^2 - F)/S = O - F$ and the fundamental group of $O - F$ is given by S. This should be intuitively clear, since every line in $\mathbf{C}^2 - F$ whose ends are identified under an nontrivial element of S, gives rise to a nontrivial loop in $O - F$. But since a flat bundle, with respect to a fixed point of reference, is given by the gauge transformations associated to every nontrivial loop starting and ending at the reference point, the map γ specifies a flat bundle and vice versa (up to gauge transformations of the bundle).

Consistency of the model requires that for each fixed point $x_0 = Dx_0$ with $Dx = rx + h$ the level matching condition (1) has to be fulfilled for the generator (r, γ_D). This translates into the requirement that the flat bundle gives rise to fractional instanton numbers satisfying (3) at every fixed point.

4 Conclusions

We have shown that the orbifolds in our class correspond to all possible flat $\mathrm{E}_8 \times \mathrm{E}_8$ bundles on the orbifold $\mathbf{C}^2/S$ with the fixed points taken out under the only restriction that the sum of the fractional parts of the gauge instanton numbers (computed from the flat bundle data via (4)) match the fractional part of the gravitational one locally at every fixed point.

Since the fractional instanton numbers are computed seperately for every E_8, this classification fully applies to M-theory on $S^1/\mathbf{Z}_2$.

Furthermore, as there exists a model for any given data satisfying the level matching conditions, these conditions should account for all global consistency conditions, at least from the ten dimensional viewpoint. This implies, by locality, that anomaly cancellation in M-theory on $S^1/\mathbf{Z}_2$ can be (and must be) fully accounted for by discussing single fixed points as in section 2.

5 Outlook

We end with a small outlook at possible uses of the computation given in [16].

One of the motivations for [16] was that the gauge shift at a given fixed point is given by a vector β which is right the amount of data needed to specify a Wilson line, say of the heterotic string on S^1. To make this similarity more explicit, consider an open ball cut out around the origin of $\mathbf{C}^2$. Dividing by $\mathbf{Z}_N$ as in section 2 we

get an A_{N-1} orbifold singularity at the origin, which can be easily blown up to a manifold M with boundary $S^3/\mathbf{Z}_N =: L_N$. This boundary is known as a lens space.

It is well known that this configuration can be found at the core of N Kaluza-Klein monopoles of the heterotic string on S^1 sitting nearly on top of each other [17]. Furthermore, taking an S^2 surrounding the monopoles in nine dimensions this sphere lifts to a bundle $S^1 \to S^2$ in ten dimensions. But the total space of that bundle is nothing but our lens space L_N: in case of one KK-monopole the S^1 bundle is well known to be the Hopf-fibration which is nothing but an U(1) bundle on S^2 with transition function $1 \in \pi_1(\mathrm{U}(1))$. Combining N monopoles just gives an U(1) bundle with transition function $N \in \pi_1(\mathrm{U}(1))$ which can be identified with L_N (see [16]).

But very far from the centers of the KK-monopoles spacetime looks locally like $S^1 \times \mathbf{R}^3 \times \mathbf{R}^{5,1}$ and the Z_N generator just rotates one time around the S^1. Therefore, associating a gauge shift β with this generator is equivalent to specifying a Wilson line given by β around the S^1.

Now one of the features of the calculation of (4) given in [16] was that for a gauge field configuration where the gauge connection A is constant around the S^1, i.e. a Wilson line, even the integer part of the instanton number can be computed:

$$I = -\frac{1}{60}\frac{1}{8\pi^2} \int_U \mathrm{tr}\, F^2 = \frac{N}{2}\beta^2 \qquad (6)$$

Even though such a configuration cannot be supersymmetric because of the sign of the instanton number, it should appear in the magnetic charge lattice of the heterotic string on S^1.

This correspondence might shed some light on the physics of the heterotic string on orbifold singularities with nontrivial gauge backgrounds.

Acknowledgements

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